Image Enhancement

SPATIAL FILTERING

$$g(x,y) = h(x,y) * f(x,y)$$

$f(x,y) \Rightarrow \boxed{h(x,y)} \Rightarrow g(x,y)$

FREQUENCY DOMAIN FILTERING

$$G(u,v) = H(u,v).F(u,v)$$

$F(u,v) \Rightarrow \boxed{H(u,v)} \Rightarrow G(u,v)$

Image Enhancement

LOW PASS FILTERING

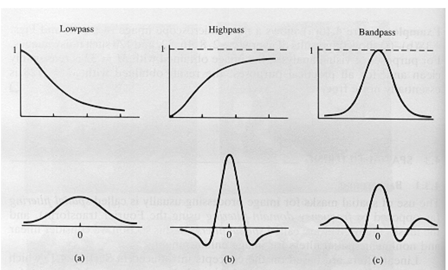
– attenuate or eliminate high-frequency components (edges and other sharp details)
– results in image bluring

HIGH PASS FILTERING

– attenuate or eliminate low-frequency components (slowly varying characteristics, such as overall contrast and average intensity)
– results in reduction of overall contrast and average intensity, and a correspondingly apparent sharpening of edges and other sharp details

Image Enhancement

Frequency domain filters $G(u,v) = H(u,v).F(u,v)$



Spatial domain filters $g(x,y) = h(x,y) * f(x,y)$

Image Enhancement

SMOOTHING FILTERS - LOWPASS SPATIAL FILTERING

- **With the assumption that image pixel values within a small neighborhood are highly correlated and that the noise components are not correlated, noise may be reduced by replacing each pixel with the *mean* over a certain neighborhood of (x, y):**

$$g(x,y) = \frac{1}{M} \sum_{(n,m)\in S} f(n,m) = \mu(x,y).$$

- **where M is the number of pixels in the neighborhood S.**
- **This is useful when only one version of the image is available.**
- **If this operation is performed over a 3 x 3 neighborhood, we have**

$$g(x,y) = \frac{1}{9} \sum_{i=1}^{1} \sum_{j=-1}^{1} f(x+i, y+j).$$

Image Enhancement

CONVOLUTION BY MASK OPERATION:
The 3x3 mean filter may be expressed by the convolution mask

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Unfortunately, the mean filter operation blurs edges and sharp features.

$$g(x,y) = h(x,y) * f(x,y)$$

Convolution mask or kernel

Image Enhancement

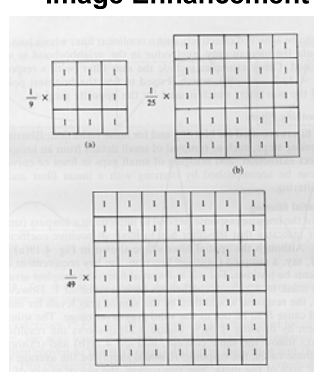


Image Enhancement

(a) Original image;

(b)-(f) results of spatial lowpass filtering with masks size of 3x3, 5x5, 7x7, 15x15, 25x25.

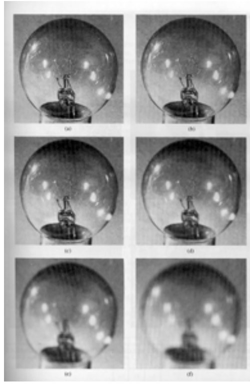


Image Enhancement

Blurring of edges may be controlled by *selective mean filtering*:

$$g(x,y)=\begin{cases}\mu(x,y), if \left|f(x,y)-\mu(x,y)\right|>T,\\ f(x,y)\; otherwise\ .\end{cases}$$

- **where T is a threshold,**
- **this is useful in *salt and pepper noise,***
- **in this applications, the central pixel at(x,y) is usually left out to use only the eight neighboring pixel in computing the mean.**

Image Enhancement

SMOOTHING FILTERS - MEDIAN FILTERING

- **non-linear filter**
- **performs better noise removal with less blurring in most cases.**

$$\begin{bmatrix}10 & 20 & 20\\ 20 & 15 & 20\\ 20 & 25 & 100\end{bmatrix} \longrightarrow \begin{bmatrix}10 & 20 & 20\\ 20 & \boxed{20} & 20\\ 20 & 25 & 100\end{bmatrix}$$

(10, 20, 20, 20, 15, 20, 20, 25, 100)

sorting: (10, 15, 20, 20, 20, 20, 20, 25, 100)

median

Image Enhancement

(a) Original image;

(b) image corrupted by impulse noise;

(c) result of 5x5 mean;

(d) result of 5x5 median filtering.

Image Enhancement

SHARPENING FILTERS - HIGHPASS SPATIAL FILTERING

- **Edge Enhancement and Extraction**
- **The gradient operator gives a measure of change in the image values in the direction specified:**

$$G(x,y)=\frac{\partial f}{\partial x}i+\frac{\partial f}{\partial y}j \quad \Longrightarrow \quad \left|G(x,y)\right|=\sqrt{\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}.$$

- **For digital, differentiation is approximated by differences:**

$$\left|G(x,y)\right|=\sqrt{\left[f(x,y)-f(x-1,y)\right]^2+\left[f(x,y)-f(x,y-1)\right]^2}, \quad or$$
$$\left|G(x,y)\right|\approx\left|f(x,y)-f(x-1,y)\right|+\left|f(x,y)-f(x,y-1)\right|.$$

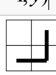


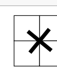
Image Enhancement

Differentiation leads to

- **removal of constant values in the direction of the operation;**
- **extraction of edges in the orthogonal direction; and**
- **removal of the average intensity (DC component).**

Roberts gradient uses cross-differences

$$\left|G(x,y)\right|=\sqrt{\left[f(x,y)-f(x+1,y+1)\right]^2+\left[f(x,y+1)-f(x+1,y)\right]^2}.$$



- **This operator computes diagonal edge gradients.The advantage of this operator is that the resulting image pixel values may be written in the same array as the input image.**

Image Enhancement

3 x 3 MASKS FOR GRADIENT OPERATIONS

Prewitt operators:

$$\frac{\partial f}{\partial x} \approx \frac{1}{3}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}; \quad \frac{\partial f}{\partial y} \approx \frac{1}{3}\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Sobel operators:

$$\frac{\partial f}{\partial x} \approx \frac{1}{4}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}; \quad \frac{\partial f}{\partial y} \approx \frac{1}{4}\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Image Enhancement

3x3 MASK FOR IMAGE SHARPENING

Laplacian: **Subtracting Laplacian:** **Unsharp Masking:**

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} -1/8 & -1/8 & -1/8 \\ -1/8 & 2 & -1/8 \\ -1/8 & -1/8 & -1/8 \end{bmatrix}$$

Image Enhancement

3X3 MASK FOR DIRECTIONAL GRADIENTS

$$0^o : \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}; \quad 90^o : \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$45^o : \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}; \quad 135^o : \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}.$$

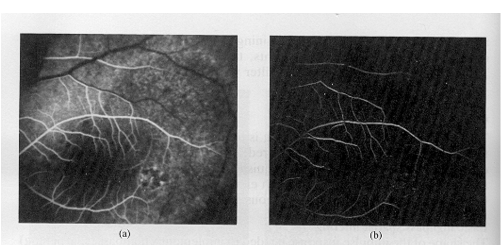
Image Enhancement

EXAMPLES OF 3X3 MASK OPERATIONS:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & -2 & -1 & -2 & 1 \\ 1 & -1 & 0 & -1 & 1 \\ 1 & -2 & -1 & -2 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 & -1 & 0 \\ -1 & 3 & 2 & 3 & -1 \\ -1 & 2 & 1 & 2 & -1 \\ -1 & 3 & 2 & 3 & -1 \\ 0 & -1 & -1 & -1 & 0 \end{bmatrix}$$

Image Enhancement



(a) (b)

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Image Enhancement

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & w & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Where w = 9A -1, with A ≥1

(a) original image;

(b) A=1.1;

(c) A=1.15;

(d) A=1.2.

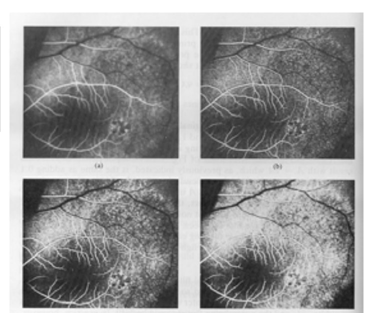


Image Enhancement

(a) original image;

(b) magnitude of Prewitt gradient;

(c) setting to 255 any gradient value over 25;

(d) setting to 255 any gradient value over 25 and setting to 0 any gradient value under or equal 25.

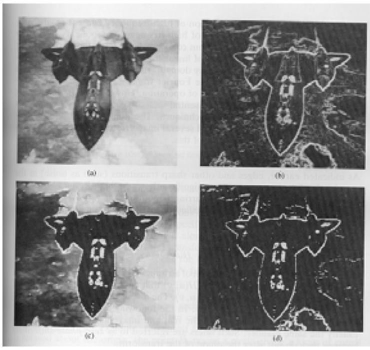
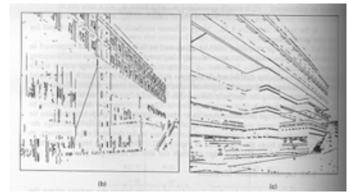


Image Enhancement



(a) original image;

(b) vertical edge detector;

(c) horizontal edge detector;

(d) Sobel edge detector;

(e) Roberts' edge detector.

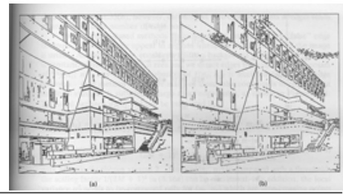


Image Enhancement

FREQUENCY DOMAIN FILTERING

- High-frequency components are associated with sharp features in the image, as well as noise.
- To achieve smoothing of images and/or noise removal, we may remove or attenuate a certain portion of the high-frequency components by lowpass filtering.

$$G(u,v) = H(u,v).F(u,v)$$

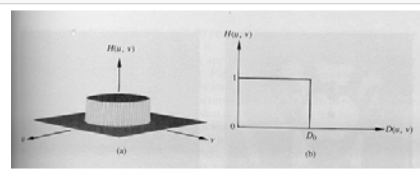
$F(u,v) \Rightarrow \boxed{H(u,v)} \Rightarrow G(u,v)$

Image Enhancement

LOWPASS FILTER FUNCTIONS:

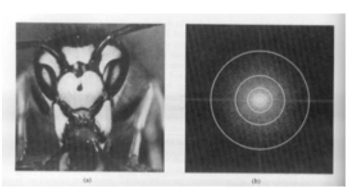
$$ideal: \quad H(u,v) = \begin{cases} 1 & if\ D(u,v) \le D_o, \\ 0 & otherwise \end{cases}$$

$$(Note: \quad D(u,v) = \sqrt{u^2 + v^2},\ i.e.,\ the\ radial\ frequency.)$$



$$G(u,v) = H(u,v).F(u,v)$$

Image Enhancement

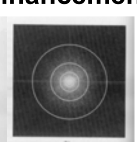


(a) 512x512 image;

(b) its Fourier spectrum with superimposed circles which radii equal to 8, 18, 43, 78, and 152 (enclose 90, 93, 95, 99, and 99.5% of the image power, respectively).

$$P_T = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} P(u,v) \qquad P(u,v) = |F(u,v)|^2$$

Image Enhancement

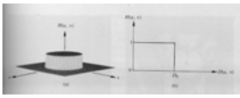


(a) original image;

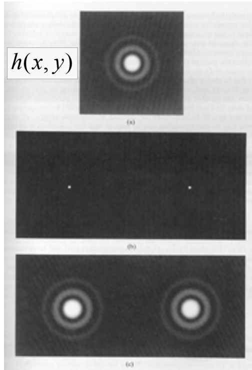
(b)-(f) results of ideal lowpass filtering with the cutoff frequency set at the radii equal to 8, 18, 43, 78, and 152, respectively.

- Ideal losspass filters results in blurring and ringing removing edge and sharp detail information of the image

Image Enhancement



$$G(u,v) = H(u,v).F(u,v)$$



$$h(x,y)$$

$$f(x,y)$$

$$g(x,y) = h(x,y) * f(x,y)$$

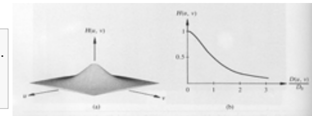
Image Enhancement

- **While "ideal" filtering is possible on computers, it is not desirable as it results in ringing artifacts around edges in the image.**
- **Exponential and Butterworth filters provide a smoother roll off, and produce smooth images with no ringing artifacts.**

Exponential: $H(u,v) = \exp\left[-\left(\frac{D(u,v)}{D_o}\right)^n\right].$ $D(u,v) = \sqrt{u^2+v^2}$

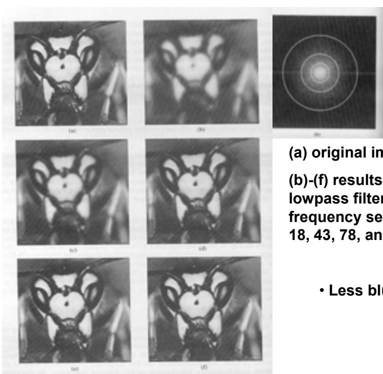
Butterworth:

$$H(u,v) = \frac{1}{1+\left[\frac{D(u,v)}{D_o}\right]^{2n}}.$$



(Note: n is the order of the filter; higher-order filters provide faster roll-off.)

Image Enhancement



(a) original image;

(b)-(f) results of Butterworth lowpass filtering with the cutoff frequency set at the radii equal to 8, 18, 43, 78, and 152, respectively.

- **Less blurring and no ringing**

Image Enhancement

(a) image digitized with only 16 gray levels (exhibits false contours);

(b) result of smothing (a) with a lowpass filter of order 1;

(c) noisy image;

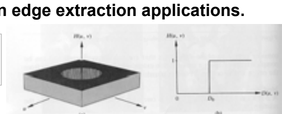
(d) results of applying Butterworth lowpass filtering to the noisy image.

Image Enhancement

HIGHPASS FILTER FUNCTIONS:

- **Highpass filters are useful in edge extraction applications.**

Ideal: $H(u,v) = \begin{cases} 1 & if\ D(u,v) \geq D_o, \\ 0 & otherwise \end{cases}$



Exponential:

$$H(u,v) = \exp\left[-\left(\frac{D_o}{D(u,v)}\right)^n\right].$$

Butterworth:

$$H(u,v) = \frac{1}{1+\left[\frac{D_o}{D(u,v)}\right]^{2n}}.$$

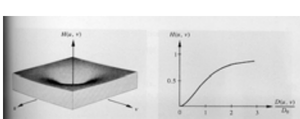
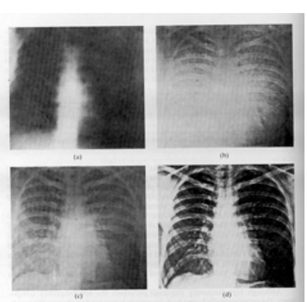


Image Enhancement



(a) original image;

(b) result after a highpass Butterworth filter (=> low-frequency components were severely attenuated, thus making different gray-level regions appear the same)

(c) result after high-frequency emphasis (high-frequency emphasis filter ≈ highpass filter + a constant);

(d) results of applying high-frequency emphasis and histogram equalization.

Image Enhancement

- Directional "sector" filters may be designed to enhance, extract, or remove features at preferred orientations, by virtue of the rotational property of the Fourier transform.
- While space domain operations affect local pixel values and features, frequency domain operations affect the image globally.
- While normally we are concerned with the magnitude spectrum to a large extent, the phase spectrum is also important. Phase has been shown to be associated with edge information to a larger extent than the magnitude of the frequency components.

Image Enhancement

HOMOMORPHIC FILTERING

$f(x,y) = i(x,y).r(x,y)$ $F(u,v) \neq I(u,v).R(u,v)$

i(x,y): illumination component (very low frequency);
r(x,y): reflectance component (medium-to-high frequency).

To separate the two components for filtering, take the logarithm:

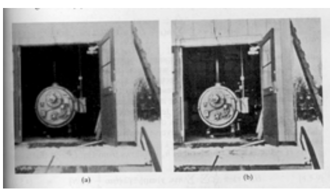
$$z(x,y) = \ln[f(x.y)] = \ln[i(x,y)] + \ln[r(x,y)]$$

$$Z(u,v) = I\ (u,v) + R(u.v)$$

$$S(u,v) = H(u,v).Z(u,v) \Rightarrow g(x,y) = \exp[s(x,y)]$$

f(x,y) ⇒ *ln* ⇒ *FFT* ⇒ *H(u,v)* ⇒ *(FFT)⁻¹* ⇒ *exp* ⇒ *g(x,y)*

Image Enhancement



(a) original image;

(b) image processed by homomorphic filtering to achieve simultaneous dynamic range compression and contrast enhancement.

(by enhancing r and suppressing i)