Digital Image Fundamentals

Digital Image Representation

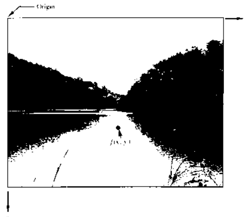


IMAGE MODEL

***f(x,y)*: a function of the spatial coordinates *(x,y)*.**

Image values *f(x,y)* usually represent physical parameters such as light intensity, density, mass, x-ray attenuation coefficient, etc. Hence,

$$0 < f(x, y) < f_{\max}$$

Digital Image Fundamentals

When imaging an object or a transparency by reflected or transmitted light, we have the *multiplicative model.*

$$f(x, y) = i(x, y)r(x, y)$$

Where $0 < i(x, y) < i_{\max}$ ***is the incident illumination function (usually uniform), and 0 < r (x,y) <1 is the reflectivity or transmittivity function* of the object imaged.**

Digital Image Fundamentals

OPTICAL DENSITY, DYNAMIC RANGE, AND CONTRAST

The *optical density OD* is the logarithm of the ratio of the intensity of light entering a point on a film I_i to the intensity of light leaving the film I_o :

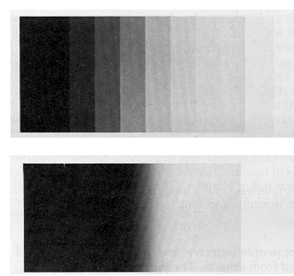
$$OD = \log_{10} \frac{I_i}{I_o}$$

The *dynamic range* of an image is $f_{\max}(x, y) - f_{\min}(x, y)$

The *simultaneous contrast* of an object region of brightness A situated on a background region of brightness B is defined as

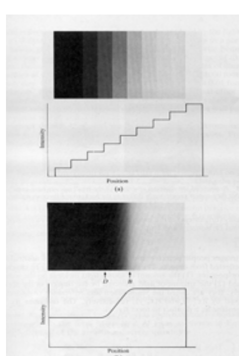
$$C = \frac{A - B}{A + B} \quad \text{or} \quad C = \frac{A - B}{B}$$

Digital Image Fundamentals

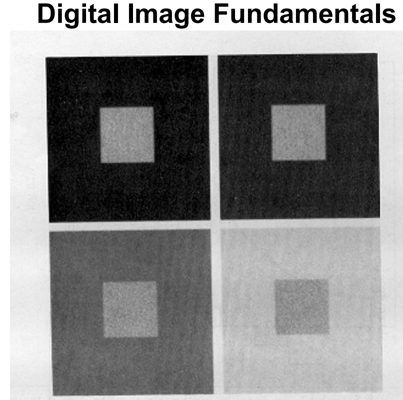


Digital Image Fundamentals

- **perceived brightness is not a simple function of intensity (Gonzalez, pp. 29)**
- **better discrimination at higher levels of illumination**



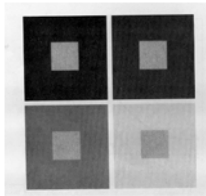
Digital Image Fundamentals



Digital Image Fundamentals

SIMULTANEOUS CONTRAST

- **all the small squares have exactly the same intensity, but they appear progressively darker as the background becomes lighter.**
  (Gonzalez, pp. 30)



- **ILLUSION & PERCEPTION LINKS**

- **dragon.uml.edu/psych/illusion.html**
- **www.yorku.ca/eye/funthing.htm**
- **www-users.cs.umn.edu/~interran/percept_links.html**
- **www.du.edu/~jcalvert/optics/ophom.htm**
- **snow.utoronto.ca/Learn2/resources/attnpcpt.html**
- **www.psych.purdue.edu/~coglab/VisLab/demos.html**

Digital Image Fundamentals

SAMPLING AND QUANTIZATION

- **For digital processing, a continuous image is converted into an array of discrete picture elements (pixels) with discrete values (gray levels).**

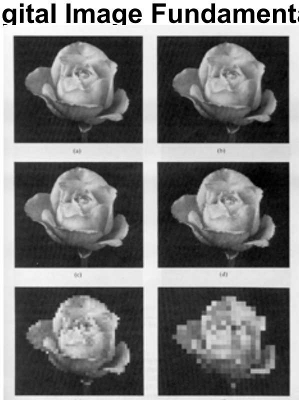
$$f(x,y) \approx \begin{bmatrix} f(0,0) & f(0,1) & \ldots & f(0,M-1) \\ f(1,0) & f(1,1) & \ldots & f(1,M-1) \\ \vdots & \vdots & & \vdots \\ f(N-1,0) & f(N-1,1) & \ldots & f(N-1,M-1) \end{bmatrix}$$

Digital Image Fundamentals

SAMPLING AND QUANTIZATION

- **In common practice, the TV raster scanning mechanism is used to break the given image into a set of rows (scan lines).**
- **An analog-to-digital converter (ADC) is then use to sample each row into a set of discrete values.**
- **Typically, *digitizing frame buffers* convert a given image into an array of 512x480 pixels, each pixel having an integral value between 0 (black) and 255 (white).**
- **Charge-coupled device (CCD) arrays are available in 2D arrays up to 2048 x 2048 and 1D arrays with 4096 or more pixels.**

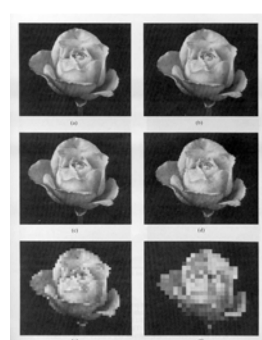
Digital Image Fundamentals



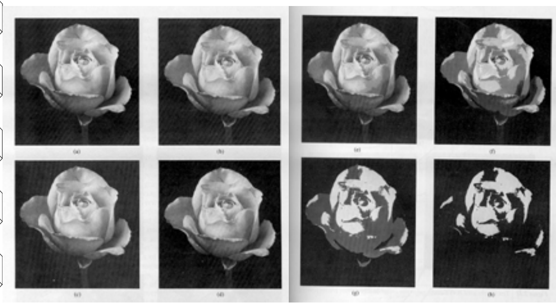
Digital Image Fundamentals

SAMPLING AND QUANTIZATION

- **effects of reducing spatial resolution**
  (Gonzalez, pp. 35)



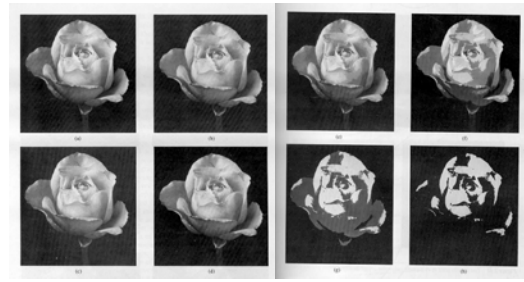
Digital Image Fundamentals



Digital Image Fundamentals

SAMPLING AND QUANTIZATION

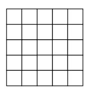
- **effects of decreasing the number of bits used to represent the gray levels in an image (Gonzalez, pp. 36-37)**



Digital Image Fundamentals

SOME BASIC RELATIONSHIPS BETWEEN PIXELS

- ***Neighbors* of a pixel *p(x,y)***
  - **horizontal and vertical neighbors: $N_4(p)$**
    (x+1,y), (x-1,y), (x,y+1), (x,y-1)
  - **diagonal neighbors: $N_D(p)$**
    (x+1,y+1), (x+1,y-1), (x-1,y+1), (x-1,y-1)
  - **$N_8(p)$: $N_4(p)$ and $N_D(p)$**

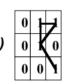


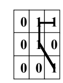
Digital Image Fundamentals

SOME BASIC RELATIONSHIPS BETWEEN PIXELS

Connectivity

- ***adjacent* pixels and satisfy a specified *criterion of similarity* (example, if they have the same gray level or a gray level of V = { … })**
  - **4-connectivity**
    - **p ∈ V, q ∈ V, q is in the set $N_4(p)$**
  - **8-connectivity**
    - **p ∈ V, q ∈ V, q is in the set $N_8(p)$**





  - **m-connectivity (mixed connectivity)**
    - **p ∈ V, q ∈ V, q is in the set $N_4(p)$**
      ***or* q is in the set $N_D(p)$ *and* $[N_4(p) \cap N_4(q)] = \varnothing$**
    - **eliminate the multiple path connections**

Digital Image Fundamentals

SOME BASIC RELATIONSHIPS BETWEEN PIXELS

Distance* between *p(x,y)* and *q(s,t)

- ***Euclidean distance*** $D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$
- ***D_4 distance (city-block distance)*** $D_4(p,q) = |x-s| + |y-t|$

4	3	2	3	4
3	2	1	2	3
2	1	0	1	2
3	2	1	2	3
4	3	2	3	4

- ***D_8 distance (chessboard distance)*** $D_8(p,q) = \max(|x-s|, |y-t|)$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

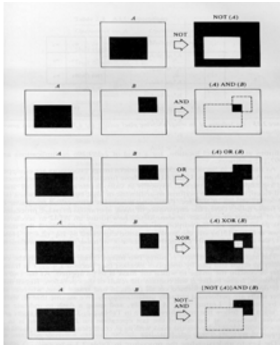
Digital Image Fundamentals

SOME BASIC RELATIONSHIPS BETWEEN PIXELS

- ***Arithmetic Operations***
  - ***Addition:*** ***p + q***
  - ***Subtraction:*** ***p - q*** ***(remove static background)***
  - ***Multiplication:*** ***p x q*** ***(correct gray level shading)***
  - ***Division:*** ***p ÷ q***
- ***Logic Operations (apply only to binary images)***
  - ***AND:*** ***p AND q***
  - ***OR:*** ***p OR q***
  - ***COMPLEMENT:*** ***NOT q***

Digital Image Fundamentals

- ***Examples of Logic Operations on binary images***
  (Gonzalez, pp. 49)



Digital Image Fundamentals

IMAGING GEOMETRY

- ***Translation***

$$X_{new} = X + X_0$$
$$Y_{new} = Y + Y_0$$
$$Z_{new} = Z + Z_0$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X_0 \\ 0 & 1 & 0 & Y_0 \\ 0 & 0 & 1 & Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{v}_{new} = \mathbf{T}.\mathbf{v}$$

Homogeneous coordinates (useful to concatenate several transformations)

Digital Image Fundamentals

IMAGING GEOMETRY

- ***Scaling***

$$X_{new} = X.S_x$$
$$Y_{new} = Y.S_y$$
$$Z_{new} = Z.S_z$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{v}_{new} = \mathbf{S}.\mathbf{v}$$

Digital Image Fundamentals

IMAGING GEOMETRY

- ***Rotation***
  - ***Z axis by an angle of θ***

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ \sin\theta & -\cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{v}_{new} = \mathbf{R}_\theta.\mathbf{v}$$

  - ***X axis by an angle of α***

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{v}_{new} = \mathbf{R}_\alpha.\mathbf{v}$$

  - ***Y axis by an angle of β***

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{v}_{new} = \mathbf{R}_\beta.\mathbf{v}$$

Digital Image Fundamentals

IMAGING GEOMETRY

- ***Example of Concatenation of transformations:***

$$\mathbf{v}_{new} = \mathbf{R}_\theta(\mathbf{S}.(\mathbf{T}.\mathbf{v})) = \mathbf{A}.\mathbf{v} = (\mathbf{R}_\theta.\mathbf{S}.\mathbf{T}).\mathbf{v}$$

Digital Image Fundamentals

- **Exercices**
- **Gonzalez pp. 77 - problem 2.13**
- **Gonzalez pp. 78 - problem 2.14**