

## ON THE REQUIREMENTS FOR SIMULATING A MEMORY EVOLUTIVE SYSTEM

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**ABSTRACT**

This paper discusses aspects of the simulation of a memory evolutive system (MES) by means of an artificial neural network. We address here the issues concerning the minimum requirements that a neuron model should fulfill in order that the network would be capable of expressing the categories underlying the MES. We conclude that Izhikevitch's formal neuron has sufficient dynamical properties to achieve these requirements.

**INTRODUCTION**

Brain and neural mathematical modeling are important issues today for neuroscience and related research areas. They help us testing ideas about microscopic and mesoscopic processes in the central nervous system. It seems tantalizing to simulate the brain in a reasonable level of accuracy; however it is realistic and desirable to construct models with increasing degree of fidelity in its capacity of describing the neural processes underlying the behavioral and cognitive phenomena.

It has been discussed that traditional models present serious limitations making them inadequate for nervous system and brain simulation [4]. These limitations range from the lacking of sufficient dimensionality to the loss of important dynamic properties [5]. Furthermore, semantic properties are usually quite difficult, if not impossible, to be considered in traditional simulations.

Category Theory [15] provides a framework for the study and simulation of complex, evolutive systems and their emergent properties and dynamic behaviors. It has been employed in the construction of networks composed of sub-networks associated to distinct sensors and operating in a suitable coordinated fashion, such that their interconnections can adapt to provide knowledge coherence. This produces a representation of semantics mapped in the network [7]. However, this model due to M. Healy (1999) [6], uses the category theory rather doing logical deductive inference,

mapping sensor primitives as symbols in a modified ART network (Adaptive Resonance Theory).

We propose, instead, to use the memory evolutive system (MES) of Ehresmann and Vanbremeersch (2007) [3], which employs category theory in a more adequate form to model perceptual processes and cognition.

**MEMORY EVOLUTIVE SYSTEM**

The basis of this model lays in a hierarchical evolutive system defined using categories. A *category* is no more than a graph (called its *underlying graph*) of nodes (objects) and links (relations) and an internal composition law on this graph, associating to each path in this graph its composite, satisfying the axioms of associativity and identity.

A graph is not a sufficient representation for complex, evolutive systems, hence categories, that are more constrained, are more expressive than graphs, as they allow the representation of complex objects comprised of more simple parts, and this is a base for the formation of hierarchy, a fundamental property in complex systems.

Taking the system at a given time, one can observe its *state category*, as the snapshot denoting how the parts of the system are interrelated at that time. This state category evolves in time by a series of complexifications implemented by partial *functors* that can add or remove nodes, join patterns of nodes and links into a more complex object or break these patterns.

The complex objects of a given level  $k+1$  (called the colimit, or inductive limit) are composed of pattern of objects belonging to levels  $\leq k$ . This process can be iterated at each complexification, generating more and more complex objects.

One can question the need for hierarchy, as the system could be reduced to the base level, ignoring the colimits. However, by the Multiplicity Principle [15], if the system contains multifold objects (having the same colimit); this leads to the formation of complex links, that are lost on a reductionist break-up of the system (see Fig. 1).

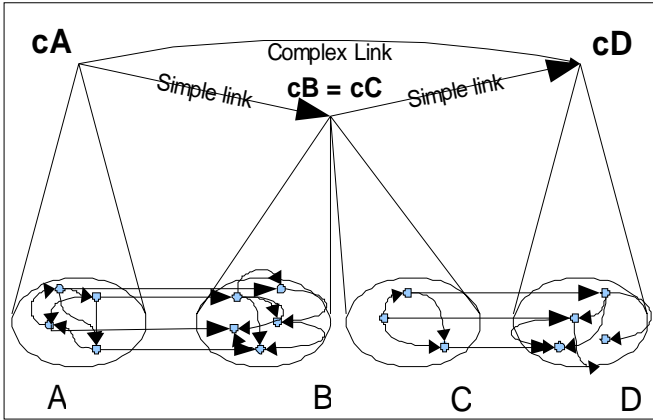


Fig. 1: The Multiplicity Principle: when two patterns have the same colimit, complex links emerge as new properties of the system

A MES can be defined as a hierarchical evolute system over a continuous time scale, with a hierarchical evolute sub-system, called the *memory*, with the same time scale that develops over time by the process of complexification. The records in the memory also develop in a hierarchical fashion, leading to more complex records and even the formation of a semantic memory.

The MES has no central regulating mechanism, instead it has a hierarchical network of partial regulatory organs, called co-regulators (CRs) that are sub-systems with specific functions and its own timing, serving to collect information, select and implement responses, and evaluate the result of these procedures, at least locally. They have differential access to the memory as they are also organized hierarchically.

The CRs operate stepwise performing successive actions:

- (a) Internal observation: formation of an internal representation of the environment, called landscape, with partial information received at that date;
- (b) Regulation: selection of objectives and admissible procedures to implement them, sending commands to the effectors and forming an anticipated landscape;
- (c) Control: at the end of each step, it evaluates the final results and takes part to their storage in the memory at the beginning of the next step;

The global procedure is the sum of the procedures of each CR. Eventually, the various CRs can have conflicting procedures, which may cause fractures in their previsions. However, higher order CRs, with longer periods of activity, have a priority when resolving conflicts between strategies.

One possible application of the MES is the study of the neural system, using the Memory Evolute Neural System (MENS), an extension of the MES imbued with special properties to deal with networks of neurons and category neurons (cat-neurons).

**CATEGORY NEURONS**

The basis for modeling the neural system lies in neurons and the synapses between them, forming an intricate graph indicating the state of the system by the activity (instantaneous

firing frequency) of the neurons and the strengths of the synapses (probability of propagating activation). Some models also include a propagation delay for the synapses.

To recognize the various features in the environment and better adapt to them, the neurons firing together can form assemblies that are patterns to represent complex mental objects. In some cases there is a coordination neuron which binds the assembly and is activated synchronously with the assembly.

However, most of the times, this neuron can't be found. In these cases, learning takes place only by reinforcing the strengths of the synapses in the assembly, according to Hebb's rule [8] or some variant. Experimental studies [2] indicate that many assemblies can lead to the same outputs, and the same item or process can activate several more or less similar assemblies depending on the context.

In the Memory Evolute Neural System (MENS), a conceptual object, called category-neuron (cat-neuron), is introduced to model the class of these assemblies activated by the same item. This multifold dynamic object can be viewed as a "higher order" neuron, or a "mental object", but it is activated by a physical event, namely the activation of any of the neural assemblies it represents (possibly non-connected). This process can be iterated obtaining cat-neurons for patterns of cat-neurons, representing synchronous hyper-assemblies.

The MENS is then defined as a memory evolute system over the lifetime of the animal, which has for the first hierarchical level the evolute sub-system **Neur**, generated by the graph of neurons, synapses and synaptic paths, where the synapses are labeled links containing the synaptic strength and the propagation delays. The higher hierarchical levels are obtained by a category of cat-neurons and the links between them (see Fig. 2).

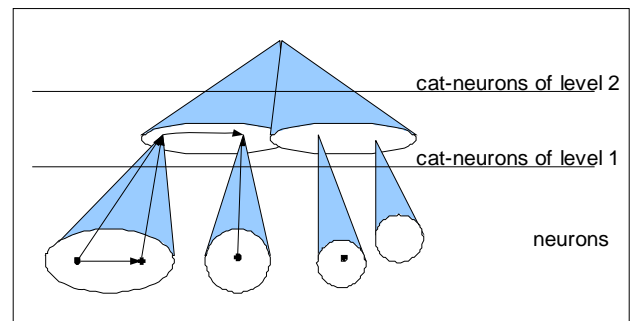


Fig. 2: Cat-neurons in the MENS

When dealing with cluster of links between assemblies, the *propagation delay of the cluster* is defined as the maximum of the propagation delays of the links in the cluster and *strength of the cluster* as an increasing function of the strengths of its links.

The strength of complex links is calculated like any composite path, by adding the propagation delays and multiplying the strengths of the simple links composing it.

## SIMULATING THE MES

The MES [3] is a mathematical model of an open self-organizing system. Concerning the MES implementation, it is necessary to choose a neuron model coherent with the dynamical aspects required for its operation. Such a neuron model have been already suggested in [3] as a cat-neuron, but in rather theoretical terms. Here we are proposing the use of a mathematical neuron dynamic model as a candidate to a computational implementation of cat-neuron and the MES model.

It is widely accepted that the Hodgkin-Huxley class of equations (or model) that describe the behavior of giant squid axon, stands as the most successful quantitative computational model in the neural science [3]. But it is also known that such model has a high computational cost, particularly for large networks. Another issue related with Hodgkin-Huxley model is that it cannot describe the functional behavior of all types of neurons found in nervous systems. There are hundreds of morphologically different neuron cells only in mammals. Along with these morphological features, neurons have physiological specializations. The cellular diversity undoubtedly underlies in the capacity of the system of forming complicated networks to mediate sophisticated behaviors [17, 1].

## THE CANDIDATE NEURAL MODEL

Recently, Izhikevitch [9, 10, 11] have presented a model of a single neuron that may represent many biophysically accurate Hodgkin-Huxley-type neural models. By treating neurons as dynamical systems, the model considers that the resting state of neurons corresponds to a stable equilibrium. Neurons are excitable because the equilibrium is near a bifurcation, and despite the existence of many ionic mechanisms of spike generation there are only four generic bifurcations of equilibria. By analyzing the phase portrait at neuron bifurcations the model can explain why neurons have many different behaviors like well-defined threshold, all-or-none spikes, hysteresis, and frequency preferences among others. As pointed in [9] these features determine the kind of computation a neuron do, not the overall ionic current per se.

A concise explanation of the model may be found in [10] and the full explanation of how the model was achieved may be found in [9]. As pointed in [m], bifurcation methodologies had enabled the reduction of neuron models to a two-dimensional (2-D) system of ordinary differential equations of the form:

$$\mathbf{v}' = 0.04 \mathbf{v}^2 + 5 \mathbf{v} + 140 - \mathbf{u} + \mathbf{I} \quad (1)$$

$$\mathbf{u}' = \mathbf{a} (\mathbf{b} \mathbf{v} - \mathbf{u}) \quad (2)$$

with the additional computational artifact of the after-spike resetting:

$$\begin{aligned} \text{if } \mathbf{v} \geq 30 \text{ mV then} \\ \mathbf{v} \leftarrow \mathbf{c}; \text{ and} \\ \mathbf{u} \leftarrow \mathbf{u} + \mathbf{d} \end{aligned} \quad (3)$$

As described in [10],  $\mathbf{v}$  and  $\mathbf{u}$  are dimensionless variables.

- $\mathbf{v}$  represents the membrane potential (scaled to millivolts);
- $\mathbf{u}$  represents the membrane recovery variable (a negative feedback to  $\mathbf{v}$ );
- $\mathbf{I}$  is a variable that represents synaptic currents or injected dc-currents.

The parameters  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ , and  $\mathbf{d}$  are also dimensionless and represent:

- $\mathbf{a}$  describes the time scale of the recovery variable  $\mathbf{u}$ . Smaller values result in slower recovery (typically  $\mathbf{a} = 0.02$ );
- $\mathbf{b}$  describes the sensitivity of the recovery variable  $\mathbf{u}$  to the sub-threshold fluctuations of the membrane potential  $\mathbf{v}$  (typically  $\mathbf{b} = 0.2$ );
- $\mathbf{c}$  describes the after-spike reset value of the membrane potential  $\mathbf{v}$  (typically  $\mathbf{c} = -65$  mV);
- $\mathbf{d}$  describes the after-spike reset of the recovery variable  $\mathbf{u}$  (typically  $\mathbf{d} = 2$ ).

Also in the above equations,  $\mathbf{v}'$  and  $\mathbf{u}'$  denote the derivatives  $\mathbf{d}/\mathbf{dt}$ , where  $\mathbf{t}$  is the time, given in milliseconds, to correspond to the other parameters.

The individual neuron model reproduces a wide range of neuronal biological behaviors such as spiking, bursting, and mixed mode firing patterns, continuous spiking with frequency adaptation, spike threshold variability, bi-stability of resting and spiking, sub-threshold oscillation and resonance, etc. But when trying to model open self-organizing systems by using such neuron model other factors must be taken into account.

More precisely, neurons must be connected one another and, by taken the biology as a model, it requires a synaptic model. It most also be taken into account that in biological world neurons generates action potential (spikes) in time, and also, it takes time to such spikes to propagate from one site to another. Furthermore, when operating together, it is expected from groups of neurons the emergence of synchronous operation, also called coalition.

The mentioned model works with a resolution of one millisecond, so time control is incorporated to the model due the nature of the spiking network to which it is connected to. Regarding to delay propagation, it is suggested in [10] that a possible extension of the model is to treat  $\mathbf{u}$ ,  $\mathbf{a}$ , and  $\mathbf{b}$  as vectors, and use  $\Sigma \mathbf{u}$  instead of  $\mathbf{u}$  in the voltage equation (1). Such procedure will account for slow conductance in multiple time scale, although the model's author warns that it would be unnecessary for networks that simulate cortices.

When coupled, spiking neurons may present patterns that resemble collective behavior as well as Poissonian patterns of firing as shown in [11].

## SYNAPTIC DYNAMICS

There are two types of synapses in the nervous systems: electrical and chemical. In electrical synapses the presynaptic and the postsynaptic terminals are not completely separated, so it behaves must like a short-circuit for spikes. By the other side, chemical synapses have a gap junction and a chemical transmitter is responsible for the continuity of propagation of the spike from pre-synaptic to post-synaptic terminals. The former presents virtually no delay on spike propagation, while the last presents typical delays of 1 to 5 ms [14].

Perhaps the most important characteristic of chemical synapses is that they can change the strength of their connection. Recent researches have shown that the strength of the connection between two chemical synapses can be modified by activity, revealed by a direct dependence on the timing of neuronal firing on either side of the synapse [12, 16, 18, 13]. The so-called STDP (spike timing-dependent plasticity) is a powerful computational characteristic of neurons because, according to the temporal delay between pre- and postsynaptic spiking activity, a connection between neurons can be strengthened (when the presynaptic spike precedes the postsynaptic one) or weakened (when the postsynaptic spike happens before the presynaptic one). Therefore, the temporal order in the precise millisecond-scale is a mechanism that provides biological neural networks with a learn system.

In [3] an STDP model of synaptic plasticity is presented into a neural network implemented with the model described above. According the authors, the dynamics of passive change of the synaptic weight  $c_{ij}$  from neuron  $j$  to neuron  $i$  are described by the second-order linear equation:

$$c''_{ij} = - (c'_{ij} - a) 10^4 \quad (4)$$

where  $a$  describes slow, activity-independent increase of synaptic weight. Such implementation is a particular one. Others forms of implementation of STDP rules may be implemented. What the example shows is that its implementation may represent another equation that probably must be calculated to each pair of synapses present into the network.

## FINAL REMARKS

The choice of the Izhikevich model for implementing the MES is founded in three issues:

- the dynamic properties of the Izhikevich model are very suitable to match the synchronization requirements implied in the MES, allowing to the arousing of the stability spans of the correlations among neural assemblies (or sub-networks, in the Healy terms);
- the existence of a rich dynamics with possible choice of diverse firing patterns that can cope with several different mappings, namely functors, between two time instances of a evolving category;
- a satisfactory trading between node complexity and topological connectivity among nodes.

This third issue refers to the number of connections that one neuron should have with the others in an assembly in order to allow sufficient capacity to the network to express the category. We are currently considering the formal characterization of this trading between the complexity or expressiveness of the node employed for the node (neuron) and the number of connections that it should make to form its neighborhood. A difficult aspect found in this formulation is that the dynamical aspects shall be considered when concerning to an evolutive model like the MES.

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